Learning Goals

1. To understand that optimum surface area and volume occur when the prism is a cube and for a cylinder when the diameter = height.

2. To be able to calculate the surface area of a cube when given its volume.

3. To be able to calculate the volume of a cube when given its surface area.

4. To be able to calculate the surface area of cylinder when given its volume.

5. To be able to calculate the volume of a cylinder when given its surface area.

6. To be able to calculate the dimensions of both a cube and cylinder.

8.8 Optimum Volume and Surface Area

Rectangular Prism

The **minimum surface area** and/or the **maximum volume** in a rectangular prism always occurs when the prism is a **CUBE**.

Volume and Surface Area Formulas for a Cube

$V = s^3$, where $s = V^{\frac{1}{3}}$ $\qquad$ V $\boxed{y^x}$ $(1 \div 3)$

$\boxed{SA} = \boxed{\cancel{6}s^2}$, where $s = \sqrt{\frac{SA}{6}}$

$\sqrt{6}$ $\quad$ $\sqrt{\cancel{6}}$

Example One

Determine the dimensions and maximum volume of a square-based prism with a surface area of 600 cm^2.

1. Plug SA into formula and solve for "s".

$SA = 6s^2$

$\frac{600}{6} = \frac{\cancel{6}s^2}{\cancel{6}}$

$\sqrt{100} = \sqrt{s^{\cancel{2}}}$

$s = 10\text{cm}$

2. Plug s=10 into volume formula.

$V = s^3$

$= 10^3$

$= 1000\text{cm}^3$

3. State the dimensions.

10 cm × 10 cm × 10cm

Example Two

Determine the dimensions and minimum surface area of a cube with a volume of 10 648 cm^3.

1. Plug in volume + solve for "s".

$V = s^3$

$(10\,648)^{\frac{1}{3}} = (s^3)^{\frac{1}{3}}$

$s = 10\,648^{\frac{1}{3}}$

$s = 22\text{cm}$

10 648 $\boxed{y^x}$ (÷3)

$s = V^{\frac{1}{3}}$

$= 10\,648^{\frac{1}{3}}$

$= 22\text{cm}$

2. Plug s=22 into SA formula.

$SA = 6s^2$

$= 6(22)^2$

$= 2904\text{cm}^2$

Cylinder

The **minimum surface area** and/or the **maximum volume** in a cylinder <u>always</u> occurs when the **DIAMETER** of the cylinder **EQUALS** the **HEIGHT**
(i.e. d = h or 2r = h).

Volume and Surface Area Formulas for a Cylinder

$V = 2\pi r^3$, where $r = \left(\frac{V}{2\pi}\right)^{\frac{1}{3}}$, and $h = 2r$

$SA = 6\pi r^2$, where $r = \sqrt{\frac{SA}{6\pi}}$, and $h = 2r$

Example Three
Determine the dimensions and maximum volume of a cylinder with a surface area of 1884 cm^2.

1. Plug in SA & solve for "r."

$\sqrt{\frac{1884}{6\pi}} = \sqrt{\frac{\cancel{6\pi} \cancel{r^2}}{\cancel{6\pi}}}$

$r = \sqrt{\frac{1884}{6\pi}}$

$r = \sqrt{}$ $1884 \div (6 \times 3.14)$ enter

$r = 10 cm$

2. Plug in r = 10 & solve for volume.

$V = 2\pi r^3$
$= 2\pi(10)^3$
$= 6283.2 \text{ cm}^3$

3. State the dimensions.

h x d , h = 2r
20 cm x 20 cm d = 2r

Example Four

Determine the dimensions and minimum surface area of a cylinder with a volume of 6400 cm^3.

1. Plug in volume & solve for "r".

$$V = 2\pi r^3$$

$$6400 = 2\pi r^3$$

$$r = \left(\frac{6400}{2\pi}\right)^{1/3}$$

* Use the radius formula.

$$r \doteq 10.06\text{ cm}$$

On your calculator...

$$(6400 \div (2\pi)) \boxed{y^x} (1 \div 3)$$

2. Plug r = 10.06 into SA formula.

$$SA = 6\pi(10.06)^2$$
$$= 1907.64\text{ cm}^2$$

3. State the dimensions.

$$h \times d \qquad h = 2r \;\&\; d = 2r$$

Complete: p. 481 # 1 - 5 (all).